

15A

1(a) $u_n = 5n$

$$u_1 = 5(1) = 5$$

$$u_2 = 5(2) = 10$$

$$u_3 = 5(3) = 15$$

⋮

$$u_{30} = 5(30) = 150$$

~~1(b)~~ (f) $u_n = 3^{n-1}$

$$u_1 = 3^0 = 1$$

$$u_2 = 3^1 = 3$$

$$u_3 = 3^2 = 9$$

$$u_{30} = 3^{29} = 6.863 \times 10^{13} \text{ (4 S.F.)}$$

15B

① (a) 4, 8, 12, 16, 20, ... (+4 to previous term)

$$\underline{u_{n+1} = u_n + 4}, \quad \underline{u_1 = 4}$$

(b) 6, 11, 16, ...
 +5 +5

$$\underline{u_{n+1} = u_n + 5}, \quad \underline{u_1 = 6}$$

(c) 7, 10, 13, 16, ...
 +3 +3

$$\underline{u_{n+1} = u_n + 3}, \quad \underline{u_1 = 7}$$

(d) 17, 13, 9, 5, ...
 -4 -4 -4

$$\underline{u_{n+1} = u_n - 4}, \quad \underline{u_1 = 17}$$

(e) 2, 4, 8, 16, ...
 $\times 2$ $\times 2$ $\times 2$

$$\underline{u_{n+1} = 2u_n}, \quad \underline{u_1 = 2}$$

(f) 80, -40, 20, -10, ...
 $\times -\frac{1}{2}$ $\times -\frac{1}{2}$ $\times -\frac{1}{2}$

$$\underline{u_{n+1} = -\frac{1}{2}u_n}, \quad \underline{u_1 = 80}$$

(g) 1, 2, 5, 14, 41, ...
 $\times 3-1$ $\times 3-1$ $\times 3-1$ $\times 3-1$
 1 3 9 27

$$\underline{u_{n+1} = 3u_n - 1}, \quad \underline{u_1 = 1}$$

15B

amount doubles each day

(2) $u_{n+1} = 2u_n$ $u_0 = 150$, (at time $t=0$ there are 150 termites)

(3) 18% drop so 82% = 0.82 left

$u_{n+1} = 0.82u_n$

$u_0 = 40\text{ml}$

(4)

$u_{n+1} = 1.005u_n + £50$

$u_0 = £100$ (initial amount)

Note: a 0.5% increase = $100\% + 0.5\%$
= 100.5%
= 1.005

(5)

15% drop so 85% left

$u_{n+1} = 0.85u_n + 40$

$u_0 = 500\text{ml}$

15C

$$\textcircled{1} \quad u_{n+1} = 2u_n - 7 \quad u_1 = 9$$

$$u_2 = 2u_1 - 7$$

$$u_2 = 2(9) - 7$$

$$u_2 = 11$$

$$u_3 = 2u_2 - 7$$

$$u_3 = 2(11) - 7$$

$$u_3 = 15$$

$$u_4 = 2u_3 - 7$$

$$u_4 = 2(15) - 7$$

$$\underline{\underline{u_4 = 23}}$$

$\textcircled{3}$ 20% reduction so 80% left

$$u_{n+1} = 0.8u_n + 40 \quad u_0 = 80$$

$$u_1 = 0.8u_0 + 40$$

$$u_1 = 0.8(80) + 40$$

$$u_1 = 104$$

$$u_2 = 0.8u_1 + 40$$

$$u_2 = 0.8(104) + 40$$

$$u_2 = 123.2$$

$$u_3 = 0.8u_2 + 40$$

$$u_3 = 0.8(123.2) + 40$$

$$\underline{\underline{u_3 = 138.56 \text{ units deep after 3 hours}}}$$

15C

$$(4) \quad u_{n+1} = 1.5u_n + 2k \quad u_1 = 4k$$

$$u_2 = 1.5(u_1) + 2k$$

$$u_2 = 1.5(4k) + 2k$$

$$u_2 = 8k$$

$$u_3 = 1.5(u_2) + 2k$$

$$u_3 = 1.5(8k) + 2k$$

$$\underline{\underline{u_3 = 14k}}$$

$$(5) \quad u_{n+1} = m u_n + c$$

$$u_1 = m u_0 + c$$

$$10 = m(5) + c$$

$$10 = 5m + c$$

$$10 - c = 5m$$

$$m = \frac{10 - c}{5}$$

$$\underline{\underline{\hspace{2cm}}}$$

$$(6) \quad u_{n+1} = m u_n + c$$

$$u_1 = m u_0 + c$$

$$5 = 2m + c \quad (1)$$

$$u_2 = m u_1 + c$$

$$23 = 5m + c \quad (2)$$

$$18 = 3m \quad (2) - (1)$$

$$\underline{\underline{m = 6}} \quad \text{sub}$$

15C ⑥ continued

$$\underline{\underline{m = 6}}$$

sub in ①

$$5 = 2(6) + c$$

$$5 = 12 + c$$

$$-7 = c$$

$$\underline{\underline{c = -7}}$$

$$\textcircled{7} \quad u_1 = a u_0 + b$$

$$30 = 10a + b \quad \textcircled{1}$$

$$u_2 = a u_1 + b$$

$$46 = 30a + b \quad \textcircled{2}$$

$$16 = 20a \quad \textcircled{2} - \textcircled{1}$$

$$\frac{16}{20} = a$$

$$\underline{\underline{a = \frac{4}{5}}}$$

sub in ①

$$30 = 10\left(\frac{4}{5}\right) + b$$

$$30 = 8 + b$$

$$\underline{\underline{b = 22}}$$

$$\underline{\underline{u_{n+1} = \frac{4}{5} u_n + 22}}$$

15C ⑦ continued

$$u_3 = \frac{4}{5} u_2 + 22$$

$$u_3 = \frac{4}{5} (46) + 22$$

$$u_3 = 58.8$$

$$u_4 = \frac{4}{5} (58.8) + 22$$

$$u_4 = 69.04$$

$$u_5 = \frac{4}{5} (69.04) + 22$$

$$\underline{\underline{u_5 = 77.232}}$$

⑧ 3.5% per month interest = $100\% + 3.5\% = 103.5\% = 1.035$

$$u_{n+1} = 1.035 u_n - 400, \quad u_0 = £4000$$

$$u_1 = 1.035 u_0 - 400$$

$$u_1 = 1.035 (4000) - 400$$

$$u_1 = £3740 \text{ owed on March 1}^{\text{st}}$$

$$u_2 = 1.035 (3740) - 400$$

$$u_2 = £3470.90 \text{ owed on April 1}^{\text{st}}$$

$$u_3 = £3192.38 \text{ owed on May 1}^{\text{st}}$$

⋮

$$u_{12} = £203.49 \text{ owed on Feb 1}^{\text{st}}$$

15C

⑧ continued

£203.49 owed on Feb 1st

$1.035 \times 203.49 = £210.61$ owed on Feb 28th

Last payment = £210.61 on March 1st, the
customer was wrong.

151)

$$a = 0.3$$

$$b = 6$$

$$(1) (a) \quad u_{n+1} = \underline{0.3} u_n + 6$$

A limit exists as $-1 < 0.3 < 1$

$$L = \frac{b}{1-a}$$

$$L = \frac{6}{1-0.3}$$

$$L = \frac{6}{0.7}$$

$$L = \frac{60}{7} \left(= 8\frac{4}{7} \right)$$

✓

$$(c) \quad u_{n+1} = \underline{-0.4} u_n + 1$$

$$a = -0.4$$

$$b = 1$$

A limit exists as $-1 < -0.4 < 1$

$$L = \frac{b}{1-a}$$

$$L = \frac{1}{1-(-0.4)}$$

$$L = \frac{1}{1.4}$$

$$L = \frac{10}{14}$$

$$L = \frac{5}{7}$$

✓

150

$$u_{n+1} = -\frac{3}{2}u_n + 8$$

1(f) A limit does not exist as $-\frac{3}{2} < -1$

1(g) $3 u_{n+1} = -u_n + 0.6$

$$u_{n+1} = -\frac{1}{3}u_n + 0.2$$

$$a = -\frac{1}{3} \quad b = 0.2$$

A limit exists as $-1 < -\frac{1}{3} < 1$

$$L = \frac{b}{1-a}$$

$$L = \frac{0.2}{1 - \frac{1}{3}}$$

$$L = \frac{0.2}{\frac{2}{3}}$$

$$L = 0.2 \times \frac{3}{2}$$

$$L = 0.3$$

$$L = \frac{3}{10}$$

15D

$$Q(2)(a) \quad u_2 = 0.8(5) + 4$$

$$u_2 = 8$$

$$u_3 = 0.8(8) + 4$$

$$u_3 = 10.4$$

$$u_4 = 0.8(10.4) + 4$$

$$u_4 = 12.32$$

$$u_5 = 0.8(12.32) + 4$$

$$u_5 = 13.856$$

$$u_6 = 0.8(13.856) + 4$$

$$u_6 = 15.0848$$

$$\underline{n=6}$$

$$(2)(b) \quad L = \frac{b}{1-a}$$

$$L = \frac{4}{1-0.8}$$

$$L = \frac{4}{0.2}$$

$$L = \frac{40}{2}$$

$$\underline{L = 20}$$

$$Q(3) \quad L = \frac{b}{1-a} \quad \begin{array}{l} a=m \\ b=6 \end{array}$$

$$20 = \frac{6}{1-m}$$

$$20(1-m) = 6$$

$$20 - 20m = 6$$

$$-20m = -14$$

$$m = \frac{14}{20}$$

$$\underline{\underline{m = \frac{7}{10}}}$$

$$(4) \quad a=k \quad b=2$$

$$L_u = \frac{b}{1-a}$$

$$L_u = \frac{2}{1-k}$$

$$\underline{\underline{\hspace{2cm}}}$$

$$a = 0.5$$

$$b = 3$$

$$L_v = \frac{b}{1-a}$$

$$L_v = \frac{3}{1-0.5}$$

$$L_v = \frac{3}{0.5}$$

$$L_v = 6$$
$$\underline{\underline{\hspace{2cm}}}$$

Limits are equal

$$\frac{2}{1-k} = 6$$

$$2 = 6(1-k)$$

$$2 = 6 - 6k$$

$$-4 = -6k$$

$$\frac{-4}{-6} = k$$

$$\underline{\underline{k = \frac{2}{3}}}$$

150

⑤ X-Pest removes 60% so 40% = 0.4 remain

$$U_{n+1} = 0.4 U_n + 450$$

$$L = \frac{450}{1-0.4}$$

$$L = \frac{450}{0.6}$$

$$L = \frac{4500}{6}$$

$$\begin{array}{r} 750 \\ 6 \overline{) 4500} \end{array}$$

$$\underline{\underline{L = 750}}$$

Pest Away removes 75% so 25% = 0.25 remains

$$V_{n+1} = 0.25 V_n + 550$$

$$L = \frac{550}{1-0.25}$$

$$L = \frac{550}{0.75} \times \frac{4}{4}$$

$$L = \frac{2200}{3}$$

$$\begin{array}{r} 733\frac{1}{3} \\ 3 \overline{) 2200} \end{array}$$

$$\underline{\underline{L = 733\frac{1}{3}}}$$

In the long term Pest Away is more effective

15D

Q6 16% leaves so 84% remains

$$u_{n+1} = 0.84u_n + 25 \quad u_0 = 50$$

$$a = 0.84 \quad b = 25$$

$$L = \frac{25}{1-0.84}$$

$$L = \frac{25}{0.16}$$

$$L = \frac{25}{\frac{16}{100}}$$

$$L = 25 \times \frac{100}{16}$$

$$= 25 \times \frac{50}{8}$$

$$= 25 \times \frac{25}{4}$$

$$= \frac{625}{4}$$

$$= 156\frac{1}{4} \text{ ml}$$

It is not safe to continue the treatment over time
as $156\frac{1}{4} > 150$ so the medicine could be harmful.

15D

$$(7) (a) u_{n+1} = 0.8u_n + 0.7$$

(80% left as 20% removed)

$$L = \frac{0.7}{1-0.8}$$

$$L = \frac{0.7}{0.2}$$

$$L = 3.5 \text{ m}$$

$$L = 3.5 \text{ m}$$

They will grow to a height of 3.5m over time.

(b) Limit of 2.5m required

$$L = \frac{b}{1-a}$$

~~$$L = \frac{0.7}{2.5-a}$$~~

$$2.5 = \frac{0.7}{1-a}$$

$$2.5(1-a) = 0.7$$

$$2.5 - 2.5a = 0.7$$

$$-2.5a = -1.8$$

$$a = \frac{1.8}{2.5} = 0.72$$

i.e. 72% remains so 28% will need to be trimmed.

150

⑧ $100\% + 2.4\% = 102.4\%$

$$u_{n+1} = 1.024 u_n + K \quad u_0 = 100$$

$$u_1 = 1.024(100) + K$$

$$u_1 = £(102.40 + K)$$

$$u_2 = 1.024(102.40 + K) + K$$

$$u_2 = £(104.8576 + 2.024K)$$

$$u_3 = 1.024(104.8576 + 2.024K) + K$$

$$u_3 = 107.37418 + 3.07256K$$

$$u_3 = 500$$

$$500 = 107.37418 + 3.07256K$$

$$\underline{500 - 107.37418} = 3.07256K$$

$$\underline{500 - 107.37418} = K$$
$$3.07256$$

$$\underline{\underline{K = 127.78}}$$